A new algorithmic model and simulation of neighboring variants for *Désordre*, György Ligeti’s first étude for piano

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**Abstract:** This article discusses a new algorithmic model of *Désordre*, György Ligeti’s first étude for piano. The model is able to accurately reconstruct the original score and automatically simulate neighboring variants. The algorithm is deterministic, that is, each set of parameter values corresponds to one and only one result. The basic strategies used to model the highly entropic aspects of the composition were the formulation of primitive versions of the secondary voices and the decomposition of rhythmic patterns at independent subjacent and explicit levels. The first parameter controls the pitch-set used by the right hand and by way of its left-hand complement. The NSP parameter (the n	extsuperscript{th} sum of different primes) determines the basic rhythmic relationships and ensures the asymmetry of rhythmic patterns. Computer simulations are further controlled by specifying units of time and pitch allowing for other, uncommonly used temperaments and divisions of the whole note. Several neighboring variants of the original piece are discussed, such as simulations of octatonic, whole-tone and chromatic scales being used as a pitch-set of one of the pianist’s hands. The piece was also simulated using other asymmetrical rhythmic relationships, as well as a temperament based on quarter-tones.

**Keywords:** Algorithmic-music; computer-aided analysis; modeling and simulation; György Ligeti; études for piano; *Désordre*.

Um novo modelo algorítmico e simulação automática de variantes para *Désordre*, primeiro estudo para piano de György Ligeti

**Resumo:** Este artigo discute um novo modelo algorítmico do primeiro estudo para piano de György Ligeti. O modelo é capaz de reconstruir precisamente a partitura original e automaticamente simular variações. O algoritmo é determinístico, ou seja, cada configuração valores paramétricos corresponde a um mesmo e único resultado. As estratégias básicas usadas para modelar os aspectos de grande entropia da composição foram a formulação de versões primitivas das vozes secundárias e a decomposição dos padrões rítmicos nos níveis independentes subjacente e explícito. O primeiro parâmetro controla o conjunto de classes de altura usado pela mão direita, e através do seu complemento, pela mão esquerda. O parâmetro NSP (a enésima soma de primos diferentes) determina as relações rítmicas básicas e assegura a assimetria dos padrões rítmicos. As simulações computacionais ainda são controladas pela especificação das unidades de tempo e altura, permitindo o uso de outros temperamentos e divisões não-usuais da semibreve. Algumas variações automáticas da peça original são comentadas, tal como simulações da escala octatônica, tons-inteiros e cromáticas sendo usadas como conjunto de classe de alturas de uma das mãos do pianista. A peça também foi simulada usando outras relações rítmicas assimétricas e também usando um temperamento baseado em quartos-de-tom.

Composed in 1985, Désordre (Fig. 1) is the opening piece of György Ligeti’s (1923-2006) first book of piano études. When first heard, the piece strikes us with its clashing sonorities and sense of coordinated chaos. The pianist materializes a play of contrasts as his left hand plays the black keys and the right hand the white keys.

Asymmetrical, accentuated melodies played over layers of unstressed, regular pulsations characterize the musical texture. Throughout the piece, superimposed melodies phase out and synchronize in cycles. Like in some of his other works, the superimposition of simple layers gives rise to intricate, unexpected patterns.

The first substantial analytic study of Désordre appeared six years after its composition\(^1\) (KINZLER, 1991). It was carried out in such a way to parallel Ligeti’s own analytical study of a 1952 piece composed by Pierre Boulez\(^2\) (LIGETI, 1960).

The 1991 study of the piece proposed that a four-stage method produced each layer of Désordre\(^3\) (KINZLER, 1991: 89-90). The first stage consisted of the definition of the primary materials, such as the rhythmic pattern and pitch series (KINZLER, 1991: 90-102). The second and third stages comprise the development of the materials through formal operations. Second-stage operations include transpositions of the pitch series and deformations of the rhythmic pattern (KINZLER, 1991: 102-105). The operations used in the third stage involved “individual shortenings or lengthenings in each hand […] to build a transition between order and disorder made musically audible by a minimal temporal shifting between the parts of the right and left hand” (KINZLER, 1991: 105). The last stage concerned “marginal changes due to the keyboard playing technique” (KINZLER, 1991: 116).

Such similarities between Désordre and Boulez’s early technique are corroborated by other authors. For instance, in his History of Twentieth-Century Music in a Theoretical-Analytical Context, Elliott Antokoletz argues that,

\[\ldots\] Désordre is dedicated to Pierre Boulez and may perhaps reflect the latter’s statement that in Le Marteau sans Maître “there is in fact a very clear and very strict element of control [but] there is also room for what I call local indiscipline”. Boulez’s notion of an “organised delirium” within a controlled, almost serialised context, is entirely applicable to Ligeti’s étude (ANTOKOLETZ, 2014: 420).

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\(^{1}\) One of the first accounts of the first six piano études was made by Canadian composer Denys Bouliane. First appearing as a radio program broadcast in April 1988, it was adapted and published in German and French the next year. It was later published in English, translated by Anouk Lang, in 2006.

\(^{2}\) Structures Ia for two pianos.

\(^{3}\) In his analysis, Ligeti summarized Boulez’s compositional method as having three working stages: main decisions, automation, and fine tuning final decisions. Ligeti would argue that such a method was not about freedom against mechanical compulsion; “automatism does not function as the counterpole to decision” (LIGETI, 1960: 36-37).
Other texts refer to Désordre from diverse perspectives: from its seeming relation to fractal geometry (STEINITZ, 1996. SHIMABUCO, 2013) to Ligeti’s roots in Balkan (BOULIANE; LANG, 2006) and Sub-Saharan traditions (TAYLOR, 2003). Eventually, an algorithmic model
surfaced⁴ (KUNZE, 2003) motivating a few papers (EDWARDS, 2011; TAUBE, 2013). The goal of such a model is to implement formal operations that would create a computer-generated reconstruction of the piece.

The most detailed study (TAUBE, 2013) tried to implement controls and parameters that “might offer a possible analog to the composer’s actual decisions” going beyond a reflection of arbitrary or implementation-dependent features of the piece (TAUBE, 2013: 276). In other words, it looked at the “knobs” and “switches” Ligeti might have manipulated to enable his “machine” to generate Désordre’s structures.

This approach proved successful as the previous works claimed to have reconstructed the piece with considerable fidelity (KUNZE, 2003; TAUBE, 2013). On the other hand, at this point, researchers were not concerned about exploring other settings for these controls. That is, no previous study show how a model of Désordre works when different decisions apply.

For instance, if we want to reconstruct Désordre by running a computer model, we must set every control to “positions” supposedly used by Ligeti. This is what previous studies aimed for. But in the event of a particular control tuned to an “inaccurate” yet valid “position”, the computer model should calculate a different composition.

Taking the above into consideration, we programmed the computer to produce a faithful reconstruction of a section of Désordre. Our goal was to reconstruct and also simulate the piece inputting a series of different values to the computer model. That is to say, we selected other materials and fed them into the same “machine”. Hence, this particular use of a model is an automated approach of producing new compositions.

Such a method would serve to answer legitimate musicological questions. For instance, we could elaborate a hypothesis about certain choices made by the composer and use computer simulations as a material basis from which conclusions may be drawn or interpretations made.

This article aims to share the following contributions:

• An analysis of the piece from a renewed perspective; namely, the understanding of its central structural features by means of parameters and formal operations (cf. section “Analysis”).
• A solution to implement the background voices in accordance with the parametric/deterministic approach (cf. section “Second and fourth voices”).
• The principle of the asymmetrical rhythmic structures that could potentially be used to analyze other pieces and stimulate new compositions (cf. section “Underlying rhythmic structure of the melodic voices”).
• An alternative model to implement Désordre’s rhythmic patterns by separating the “generative” underlying structures from the hardcoded “surface level” transformations (cf. section “Durational perturbations and form”).

⁴ Kunze developed the algorithmic model as part of a Ph.D. exam at Stanford. The internal report describing the Common Music 1.3 program was written in 1999, revised in 2003, and later reworked by Taube (2013).
• The specification of parameter values for reconstructing the piece and its subsequent, substantial manipulation (cf. sections “Simulation” and “Conclusions”).

To achieve these results, our working method consisted of approaching musical dimensions, primarily pitch and durations, as numerical sequences. Pitch is represented using the MIDI standard and rhythm as multiples of a reference, the beat unit, which in turn is expressed as a division of the whole note. In this context, to model the musical score implied the need to discover and relate formal processes that revert the “numerical data” to progressively more compact and simpler stages. Implementing certain aspects either as parameters or constants depended on investigative matters, inquisitiveness, and technical impositions

Where appropriate, formal procedures have been described as numerical formulae throughout the paper. A more complete and comprehensive description of the algorithm is provided in Appendix 1.

We used OpenMusic visual programming language (BRESSON; AGON; ASSAYAG, 2011). It allowed us to code custom instructions using the Common Lisp programming language (STEELE, 1990) and to promptly represent the simulations using traditional music notation while generating MIDI files.

Analysis

Layers of texture. Four voices, or layers, constitute the musical texture of the piece. Each hand plays two layers simultaneously. These are interdependent, so that certain qualities are achieved when they are superimposed. It includes, but is not limited to, twelve-tone complementarity and texture perspective (with foreground and background layers).

We identify the first and third voices as the foreground, accentuated melodies. The score emphasizes the voices through consistent forte dynamics and octave doubling (Fig. 2, the layers are highlighted in green and blue, respectively).

5 When modeling a musical score, the decisions involving some parameters can be determined by the composer’s style or by the specific attributes of the instruments for which the piece was composed. For instance, in the case of the piece’s pitch-unit (its smallest pitch interval), the semitone is virtually an imposition of the instrument. We say “virtually” because even though the piano doesn’t allow a pitch-unit smaller than the semitone, it theoretically allows a larger unit, like the whole tone. In the same way, the division of the chromatic scale into five and seven notes is strongly suggested by the very design of the piano. However, we can speculate that other divisions of the chromatic space can provide combinations of pitch-set plus complement that are equally interesting from a creative or theoretical perspective. Finally, the parameters described below could be implemented as constants and vice versa adding significant subjectivity to the work and allowing other conceivable implementations based on the analysis which follows.

6 This is supported by the composer’s drafts where Ligeti lays out four voices in four different pentagrams. While we did not have access to these drafts, there is a fragment of one on the cover of the facsimile edition (LIGETI, 1986). On this fragment, it is also possible to see what could be assumed to be working titles of, or keywords for, the piece. These include: Mechanism, pulsation, décalage, ‘irregularité’, mouvement, Détraquement, along with the final title and other keywords (cf. also footnote 38 in KINZLER, 1991: 105).

7 The layers are classified from top to bottom, according to their rank in the piano system of the score.

8 Note that in some points the octave doubling observed in the foreground voices gives place to different voicings. This means that these voices embody simultaneous intervals other than the octave. Additionally, the notes of either the first and third voices may occasionally be executed without any doubling or parallel intervals. Eventually, in the final pages of the score, these voices metamorphose into a “melody of chords”,...
Fig. 2: First measures of Désordre's musical score showing the different voices highlighted by different colors. The effect of a texture in layers (foreground vs. background) is reinforced by the fixed dynamic markings assigned to each voice.

Spread over an uninterrupted flow of eighth notes, alternating descendent skips and ascending stepwise movement embody the second and fourth voices. These voices serve as background layers and are de-emphasized through consistent piano dynamics. These voices give the texture a lively, active, toccata-like character (Fig. 2, layers highlighted in orange and red, respectively).

The pitch-collection used by the first and second voices (the right hand) is the seven white keys of the piano keyboard, the diatonic scale. The collection used by the third and fourth voices (the left hand) is the five remaining (black) keys, i.e., the complement\(^9\). Additionally, from each collection, specific pitches (on occasion, B and D\(^\#\)) were chosen as the roots of the right- and left-hand layers\(^10\).

The choice of a particular pitch-collection from which a twelve-tone complement is calculated is the first decision associated with a formal operation that we highlight on the score. We can combine the choice of such a pitch-collection, root, and respective octave placement in a single parameter. We will arbitrarily call this parameter the gamut. The relationship between the gamut and the right- and left-hand pitch-sets can be expressed with reminiscent of Conlon Nancarrow’s Études for Player Piano. On account of these features, one could say that these parallel intervals, including the octave doubling, is the result of two voices playing together instead of just one (KINZLER, 1991: 90-91). The different voicings, aside from the octave doubling, are nevertheless not prominent in the first section, which is the only section we intended to reconstruct. Therefore, they were not put through our analysis or incorporated by our model.

\(^9\) It may appear that each foreground voice has a unison relationship with every other few notes from their respective background voices. Despite that, within just a few measures this pattern is broken.

\(^10\) Which gives B a locrian mode and D\(^\#\) minor a pentatonic mode.
the usual set-builder notation as \( RH = \text{Gamut} \) and as shown in Equation I, where \( X \setminus Y \) is the set-theoretic complement and pitches are represented in midicents\(^1\).

\[
LH = \frac{6000}{\text{middle-C}} + \{ x : (\exists k \in \mathbb{N}) [x = k \cdot \text{pitch-unit}, x < 1200] \} \\
\setminus \{ x : (\exists k \in \text{Gamut}) [x = \text{mod}(k, 1200)] \}
\]


**Equation I**

The choice of a **doubling**, viz. a fixed-pitch interval that colors and reinforces a melodic voice, is the second parameter we highlight. Its manipulation, however, has a weaker influence on the simulations of the piece. We can say then that the gamut is a **stronger parameter**, while doubling for its ornamental nature is a **weaker parameter**\(^2\).

**Underlying rhythmic structure of the melodic voices.** Each of the first and third voices has an individual melody and is articulated in different pitch scales (modes). Although both melodies are similar, their recurrent, inexact, and superimposed repetition gives rise to an intricate texture. The **first voice’s melody** has 26 notes and lasts 14 measures, after which it is consistently repeated and transposed one diatonic degree up. The **third voice’s melody** has 33 notes and lasts 18 measures, after which it is consistently repeated and transposed two pentatonic degrees down.

The iteration of each melody constitutes a cycle. Apart from the beginning, the cycles from each hand coincide twice, delineating the three sections of the piece\(^3\). Although each section and cycle has unique properties, they share the same underlying rhythmic and pitch structures.

We can uncover the anatomy of these melodies by breaking them up into smaller segments like phrases and rhythmic cells, and segmenting and classifying them by following the conventions of previous studies (see KINZLER, 1991: 97. BOULIANE; LANG, 2006: 168).

We will thus refer to the underlying cycle of each melodic voice as the **diatonic melody** and **pentatonic melody**. Each of these melodies can be broken into three phrases. Melodic cadences delineate them, except for one, as we will see below.

Different orderings of three different **rhythmic cells** form the phrases. A rhythmic cell encompasses one or more durations, measured in beat units. The first rhythmic cell, let us name it “A”, contains two durations. The first duration is equal to three beat units and the second is

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\(^1\) The midicent is the MIDI number times one hundred. It was devised to deal with microintervals, thus avoiding decimal MIDI numbers. Its use is required by the OpenMusic environment. Later on, we comment on a simulation making use of microintervals.

\(^2\) The first pages of the score suggest only a weak interdependence between the foreground octave doubling and the respective “background ambitus”, an apparent correlation which is progressively broken as the piece goes on. Following the conventions of previous studies, we cannot support this association with any kind of formalism. For this reason, we classify the doubling as a weaker parameter, which would not be the case if it were formalized as having an effect on the background or any other structural feature of the voice.

\(^3\) Our three-section segmentation follows the conventions described by Taube (2013) and therefore contrasts with that of two segments proposed by Bouliane and Lang (2006).
equal to five. The second cell, “B”, is the reversion of cell “A”. That is to say, a duration equals five units followed by another of three. The last rhythmic cell, “C”, works as a resolution. It encompasses a single duration lasting eight units. In summary, a phrase is the alternation of cells “A” and “B” that may end with cell “C”.

The first phrase of both diatonic and pentatonic melodies is the succession “A”, “A”, “B”, and “C”. In the case of the original values, it can be more directly represented by the tuple\(^{14}\) \((3, 5, 3, 5, 3, 8)\). We denominate this tuple as phrase \(A\).

The second phrase of both melodies, diatonic and pentatonic, has the same rhythmic structure. They differ from the first only in their pitch profiles. For this reason, we term this sequence as phrase \(A'\).

On the other hand, diatonic phrase \(B\) does not end with the concluding cell “C”. It causes the ensuing reiterations of the diatonic melody to be less anticipated, more sudden. We have the sequence AABABA as the diatonic third phrase and AABAAABABC as the third phrase of the pentatonic melody. Therefore, phrases \(B\) consists of freer orderings of cells “A” and “B”. They also differ by their longer extension of five and seven notes.

\[
\text{Fig. 3: Segmentation of phrase A down to its rhythmic cells: A (3+5), B (5+3) and C (8). Those rhythmic cells appear unchanged in cycle 11 of the first voice and cycle 1 of the third voice, as shown on the bottom of the image.}
\]

The divisions of a given duration (or a given measure) into groupings of 3, 5 and 7 units\(^{15}\) are connected to what Simha Arom calls Ligeti’s “rhythmic oddity” or pseudo-aksak: “globally symmetrical figures that you can never segment into two equal parts” (AROM, 2011: 115-116). In this way it could also be seen as a play with prime numbers (BAUER, 2011: 85). In fact, rather than being arbitrary values, those durations express a very particular relation. As the relation between the pitch-sets of right and left hand is central to the character of the piece, so is the relation between those durations. Should we simulate the score with another set of rhythmic cells, we

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\(^{14}\) In this article we use the following convention: whenever a set (an unordered collection) is described, we use curly brackets; parentheses are used for (infinite) sequences and angle brackets for finite sequences, also called here as tuples or series; square brackets are used whenever a sequence is treated as a vector or matrix.

\(^{15}\) Eight units become seven in the final score (more on that below).
should do it in such a way that cells A and B contain different primes whose sum corresponds to cell C\(^{16}\). That is, \(\text{durations} = \{(x, y, z) \in \mathbb{P} \times \mathbb{P} \times \mathbb{N} : x \neq y \land x + y = z\}\), which implies that cells \(A = \langle x, y \rangle\); \(B = \langle y, x \rangle\); \(C = \langle z \rangle\) where \(x, y, z \in \text{durations}\).

For instance, both the tuples \((2, 3, 2, 3, 3, 2, 5)\) and \((5, 7, 5, 7, 7, 5, 12)\) satisfy the above constraints and are then considered valid alternatives for the original sequence. On the other hand, the tuple \((6, 10, 6, 10, 10, 6, 16)\), an “augmented” version of its original \((2x)\), does not satisfy those constraints described above and, therefore, would not be a valid alternative.

We can control this feature with a parameter that we arbitrarily call \(\text{NSP}\) (the \(n\)th sum of two different primes). When \(\text{NSP}\) is 0, the integer 5 is chosen to be decomposed into primes 2 and 3. When \(\text{NSP}\) is 1, the integer 7 is chosen to be decomposed into primes 5 and 2, and so on\(^{17}\). It can be described by the function \(\text{nsp}(n) \mapsto (5, 7, 8, 9, ...)_n\).

After a given integer is chosen by \(\text{NSP}\) and decomposed into two prime numbers, the prime numbers are then arranged in such a way as described above (Fig. 3) to form the phrases A, A’ and B for both melodies.

**Pitch structure of melodic voices.** Both diatonic and pentatonic melodies are built on two monolithic series, a 26-pitch diatonic series and a 33-pitch pentatonic series, respectively. The pitch profile of the diatomic melody, in particular, seems to be reminiscent of Eastern European folk-like melodies of the slow movements of Ligeti’s *Sonatina* (1950) and the *Violin Concerto* (1990-1992).

On the other hand, similar to their rhythmic counterparts, the melodic voices indicate an underlying, regular pattern upon which a second process further develops them\(^{18}\). As the pitch sequences move forward, the melodic pattern becomes more complex. Their ambitus grows, while different intervals are introduced. The pitch profiles outline a sinusoidal shape increasing in amplitude.

The pitch profiles of both melodies also appear to confirm a three-phrase segmentation as shown above. For instance, the diatonic melody’s phrase A comprises the first seven pitches within a fourth. The next phrase, A’, is also a variation of the first phrase, comprising the seven following pitches within a major sixth. The third phrase, B, comprises the last twelve pitches ranging from A4 to A5 (Fig. 4). The beginning of each phrase is indicated by a characteristic repeated tone (cf. BOULIANE; LANG, 2006: 168).

\[\text{16}\] We denote the set of prime numbers as \(\mathbb{P}\) and, later on, the set of positive integers as \(\mathbb{Z}^+\).

\[\text{17}\] The sequence of integer numbers that \(\text{NSP}\) maps to is referenced as A038609 on the On-line Encyclopedia of Integer Sequences.

\[\text{18}\] For instance, phrase A’ could be constructed as a repetition of phrase A, but with a larger ambitus as its parameter that comprehends two octaves. Once this predictable, periodic, ideal structure has been built, a second algorithm would subtly disturb it, finally generating an ambiguous pattern that, while it is not random, is not completely periodic. Furthermore, such a strategy would endorse Amy Marie Bauer’s interpretation of the piece which assumes Ligeti added deliberate “mistakes” or “random variations” at many levels of the piece (BAUER, 2011: 87).
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The pitch profile of the pentatonic melody is similar to the diatonic being described as its “distorted pentatonic mirror” (BOULIANE; LANG, 2016: 12). Its phrase A and A’ also contains seven notes, but within a fourth and a minor seventh, respectively. Phrase B comprehends nineteen notes within an even wider range of D₃ to G₄ (Fig. 5).

We represent the pitch structure of the diatonic and pentatonic melodies as a series of degrees. This means, we classify each pitch according to its distance, stepwise, from the starting position of the mode it is based upon. As a matter of fact, every voice is represented in this manner (see below, the 2nd and 4th voices). It allows us to maintain the relative pitch structure of the piece while the parameters (like gamut and pitch-unit) are manipulated from simulation to simulation.

The mapping of any given degree \( x \) – be it simple \( x < n \) or compound \( x > n \) or \( x < 0 \) – to a given tuple \( S = (a_0, a_1, ..., a_{n-1}) \) where \( n \) is the length of \( S \) and \( \forall a \in LH \land RH \), can be obtained by the function \( \delta_S : \mathbb{Z} \rightarrow \mathbb{N} \) defined by the mapping described in Equation 2, where \( \lfloor x \rfloor \) is the floor function.

\[
\delta_S(x) \mapsto a_{\text{mod}(x,n)} + 1200 \lfloor \frac{x}{n} \rfloor
\]

**Equation 2**

**Second and fourth voices.** If the ascending stepwise motion and the descending skips are recurring features of the second and fourth voices, their underlying series of pitches do not seem to have a clear repeating pattern that would allow for a structured segmentation. Also, it appears there are virtually no clues left that would allow one to retrace a rule-based
compositional process behind the voices\textsuperscript{19}. These voices can relate to what one study (BAUER, 2011: 87) calls a “non-localizable” and “non-reversible” compositional result. Another author (KINZLER, 1991: 119) seems to suggest that the second and fourth voices are only suitable for an analysis of their more “global” principles. A previous computer model (KUNZE, 2003. TAUBE, 2013: 287) simulated both voices by means of a pseudorandom number generator procedure\textsuperscript{20}.

That being the case, it appears it may not be practical to elaborate formal schemes that, in a deterministic approach, would model the second and fourth voices. On the other hand, it is possible to at least simplify both voices and reconstruct the predictable aspects of their “surfaces” with an algorithm.

Our solution consists in the conception of a “primitive”, compressed version for each voice. These primitive versions are stripped of all stepwise motion. Here, “anchor”, or pivot notes, last longer than one beat unit. They are numerically represented by a series of degrees and a series of durations. For instance, the first elements of the second voice’s primitive series of degrees are \(0, 3, 0, 3, 1, 5, 0, 3, 2, 1\). The first elements of the corresponding series of durations are \(1, 2, 1, 4, 1, 2, 1, 4, 5, 1\). From that, an algorithmic process replaces the pivots by an equivalent ascending stepwise motion based on the corresponding degrees (Fig. 6).

\[\text{Fig. 6: Illustration of the second and fourth voices. Simplified versions of the voices are represented as a series of degrees and a series of durations. They are subjected to a process that reintroduces the ascending stepwise motion, reconstructing the original voices.}\]

The reconstitution of any “primitive” pair of duration and degree into the original stepwise movement can be modeled by the function \(\text{stepwise}: \mathbb{Z}^+ \times \mathbb{Z} \to \mathbb{R}^n \times \mathbb{N}^n\) defined by Equation 3, where the \(\delta\) function is defined in (Eq. 2) and the beat unit is represented as a fraction of the whole note, that is \(\frac{1}{8}\) for the eighth note.

\textsuperscript{19}Again, at least for the first section, the impression is that the octaves of the foreground voices influence the ascending patterns in the background. Nevertheless, beyond that, little is known that would allow for a “structural” reconstruction.

\textsuperscript{20}The programmer referred to the procedure as “fake-background” (KUNZE, 2003).
We should note, nevertheless, that this simplification does not seem to have a significant influence (in comparison to hardcoding the entire background voices) on the simulations. It could be useful, however, should one want to implement the direction of the stepwise movement (upwards or downwards) as a parameter or experiment with non-fixed pitch and beat units.

**Durational perturbations and form.** In addition to acting on each cycle with a different transposition, Ligeti systematically transforms the durations of the rhythmic pattern of both diatonic and pentatonic melodies. At practically every new cycle, a separate process affects durations with greater or minimal intensity according to an expressive compositional plan.

That process seems to induce the perception of hand-independent tempo variations, as noted in Boullane and Lang (2006: 170), Kinzler (1991: 99), and Taube (2013). It works by subtracting beat units from the actual written rhythm. As a result, a strong effect of desynchronization is achieved. These variations can emulate the traditional tempo markings of *accelerando*, and *allargando* (by adding beat units instead) and may be better described by the French words found on the sketches, *déraquement*, *contraction* and *dilatation*\(^{21}\). For instance, phrase A and A’ of the first diatonic cycle have one eighth note subtracted from their last rhythmic cell (8 units become 7). This causes the diatonic melody to lag behind the pentatonic even before their first iteration.

The separate process can be represented by another numeric sequence, a vector, indicating how many units should be subtracted, if any, from the rhythmic pattern. The change in phrase A of the first diatonic cycle—right hand—can then be represented by the vector \([0 \ 0 \ ... \ -1]\) (That is, only the last duration is one unit shorter). The “rapid acceleration” of phrase B cycle 5 of the same voice is represented as \([-2 \ -3 \ -2 \ -3 \ -3 \ -2 \ -5]\) (Fig. 7). The *allargando* in phrase A of the pentatonic 11th cycle is represented as \([0 \ 2 \ 0 \ 3 \ 4 \ 0 \ 5]\). The final, written rhythmic figures may be expressed (in \(\mathbb{R}^7\)) by a component-wise vector addition in the form \(\mathbf{u} + \mathbf{v} = [u_1 \ u_2 \ ... \ u_m] + [v_1 \ v_2 \ ... \ v_m] = [u_1 + v_1, u_2 + v_2, \ldots, u_m + v_m]\), where \(\mathbf{u}\) is the underlying series of rhythmic values (Fig. 3), \(\mathbf{v}\) is a series of perturbation values and \(\|\mathbf{u}\| = \|\mathbf{v}\|\) is implied.

\(^{21}\) Previous studies, especially Kinzler (1991), clearly differentiate between the “punctual” alterations from the constant shortenings and lengthenings of the rhythmic patterns as they occur in the second section.
As the diatonic and pentatonic melodies have different lengths, each section has a different number of diatonic and pentatonic cycles. If the cycles were not transformed by the subtraction of their durations, they would synchronize after seven diatonic and nine pentatonic cycles (incidentally, two prime numbers), adding up to 1008 beat units.

Instead, such transformations allow the voices to synchronize much earlier. Then, for the first section we have 3 diatonic and 4 pentatonic cycles. The first section totals 404 beat units, after which the voices synchronize, precipitating the second section. The second section consists of 5 diatonic and 6 pentatonic cycles throughout the course of 231 units. The voices synchronize again at the start of the third section. It also comprises three diatonic and four pentatonic cycles during 429 units.

Each of the three sections can be categorized, according to their implicit pace, in the following way:

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<thead>
<tr>
<th>First section</th>
<th>Second section</th>
<th>Third section</th>
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<tbody>
<tr>
<td>Irregular then contraction</td>
<td>Contraction</td>
<td>Regular</td>
</tr>
<tr>
<td>Regular then contraction</td>
<td>Contraction</td>
<td>Dilatation</td>
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</tbody>
</table>

Table 1: Each section of Désordre classified by the intended simulated tempo using the actual written transformation of the underlying rhythmic pattern. The first row refers to the right hand while the bottom to the left hand.

Fig. 8 shows the form of the first section of the piece for both hands through the iteration of the rhythmic pattern.
Fig. 8: Diagrammatic representation of Désordre’s first section. The entire section lasts 409 eighth notes that are divided into four cycles in the case of the first voice, and three cycles, in the case of the third voice. Each cycle of each voice is affected by a different transposition shown by the two respective piano rolls drawn in the picture. The barcodes represent the three different rhythmic cells, shown in white, grey and black, in the succession in which they appear in the original score. Their compression in the last cycle of each voice indicates the rapid acceleration that distorts the rhythmic pattern at the end of the section.
Simulation

From the procedures described above we were able to reconstruct the first section of the piece for each voice. The overall algorithm is described in Appendix 1. The model was built around six parameters; the first one which we called the gamut, that is, the set of pitches in which the first two voices are based; the NSP, i.e., the \( n^{th} \) sum of primes (an integer index), the doubling-interval which adds a constant, fixed interval to the first and third voices; the beat-unit; the tempo, and finally the pitch-unit. The reconstruction is calculated when the simulation is run using a specific set of parameter values. They are described in Table 2 and an excerpt from the output represented in traditional notation as produced by the OpenMusic environment is presented in Appendix 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Reconstruction value</th>
<th>Impact</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gamut</td>
<td>(7100 7200 7400 7600 7700 7900 8100)</td>
<td>Strong</td>
<td>List (midicents)</td>
</tr>
<tr>
<td>NSP</td>
<td>2</td>
<td>Strong</td>
<td>Integer (positive)</td>
</tr>
<tr>
<td>Doubling-interval</td>
<td>-1200</td>
<td>Weak</td>
<td>Integer (midicents)</td>
</tr>
<tr>
<td>Beat-unit</td>
<td>1/8</td>
<td>Weak</td>
<td>Number (fraction)</td>
</tr>
<tr>
<td>Tempo</td>
<td>252</td>
<td>Weak</td>
<td>Integer (beat units per minute)</td>
</tr>
<tr>
<td>Pitch-unit</td>
<td>100</td>
<td>Strong</td>
<td>Integer (midicents)</td>
</tr>
</tbody>
</table>

Table 2: Each parameter of the final computer model of \( \text{Désordre} \). The parameters can be classified as strong, i.e., those that have a powerful impact on the structural features of the piece, or weak, those that can only transpose structures or change the manner of its notation. In addition, the table shows the type of each parameter and the specific values needed to recalculate the original piece.

Parameter: gamut. As we saw before, the original piece reveals an antagonism between a seven-note and a five-note scale. In this simulation, an eight-note scale was used, namely, the symmetrical octatonic scale. Its complementary set is the four-note diminished-seventh chord (also symmetric). Fig. 9 presents the beginning of this variation in traditional notation, while Fig. 10 shows the piano roll of its entire first section.

The resulting simulation, with the other parameters set to those of the original piece, has a harmonic “steadiness” that contrasts significantly with the original piece. While the twelve tones of the chromatic scale are similarly laid out in the overall texture, there is a significant change in intervallic quality and harmonic motion. There are only two octatonic modes through which the right-hand melody can be expressed, and the third voice is limited to the “non-modally-transposable” complement.

In addition, the harmonic prominence of the tritone and major thirds of the original version is replaced by a configuration where the perfect fourth and fifth is more frequent. Finally, because the diminished-seventh chord composes pitch-collections of both hands, its constituent intervals (m3, TT, and M6) cannot occur harmonically between hands.
In the second simulation, in place of the original diatonic set, the also symmetric whole-tone or hexatonic scale was used. The complement set of a whole-tone scale starting in C is another whole-tone scale rooted on C#. It means that both hands have the same number of pitch-classes in their respective sets (6+6).

The immediate consequence of this choice has to do with the transpositions occurring after each cycle. As the whole-tone scale has a major second as its smaller (and single consecutive) interval, the reiterations of the first- and third-voice melodies develop a broader ambitus, reaching the outer extremities of the keyboard considerably faster.

Secondly, its choice allows for one of the most uniform and unvaried results from a pitch-domain perspective. For instance, besides resulting in the same complement, the whole-tone scale cannot be articulated in more than five melodic intervals\(^\text{22}\). When considering the harmonic encounters between hands, either the foreground melodies or background runs, there are only six possible harmonic intervals\(^\text{23}\).

Moreover, the hexatonic scale has only one mode, which means that the foreground melodies are continuously transposed without changing their internal intervallic structure giving each cycle a sense of sameness. These properties account for a very “static” textural quality, which may allow the ear to focus on other dimensions of the piece like rhythmic patterns and voice coordination. Fig. 11 shows the beginning of this variant in traditional notation while Fig. 12 shows the piano roll of the first section.

\(^{22}\) M2, M3, TT, m6, m7.

\(^{23}\) m2, m3, P4, P5, M6, 7M.

Fig. 9: First measures of the simulation using an octatonic scale as the pitch-set for the right hand.

Fig. 10: Piano roll (MIDI note vs. time) of the first section of the respective simulation using an octatonic scale as the input of the gamut parameter.
In the last test for the gamut parameter, something more unusual was tried as an example of extreme values. This time, the pitch-set chosen for the right hand is a ten-step chromatic scale. In the twelve-tone pitch-space, it only yields a two-note complement for the left hand. This configuration has an even stronger effect on the development of the ambitus of both hands. In the case of the first voice, as there are more semitones to cover inside one octave, its ambitus becomes very narrow. On the third voice, the exact opposite happens as there are only two pitches inside the octave; every two degrees are an octave apart. The ambitus of the third voice develops significantly faster. Fig. 13 shows the beginning of the resulting variation and Fig. 14 shows the piano roll of its first section.
Fig. 13: Excerpts of the resulting score of a simulation using a ten-step scale as the pitch-set for the right hand.

Fig. 14: Piano roll of the first section of the respective simulation using a whole-tone scale as the input of the gamut parameter.
Parameter: NSP. The NSP parameter influences the rhythmic pattern played by the first and third voices. By manipulating it, the proportions between the “melodic voices” and the “toccata-like voices” can greatly change. Additionally, irrational rhythms can arise from the relation between different time signatures and the fixed metric of the second and fourth voices. This parameter can strongly alter the perception of the piece, sometimes creating the impression of very strong desynchronization. As the NSP-value gets higher, the fixed additions to selected durations of the rhythmic pattern have less and less effect. To illustrate the manipulation of this parameter, three different instances are presented. In the first case we have the value NSP = 0, where the number five is chosen to be decomposed into primes two and three. Here the background voices will play eight eighth notes instead of ten in the first measure (5/8 time), and the durations of the rhythmic pattern become closer (2, 3 and 5). For instance, in the case of the first cycle of the first voice, we have the following written rhythms:

\[
A = [2 \ 3 \ 2 \ 3 \ 3 \ 2 \ 5] + [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -1] = [2 \ 3 \ 2 \ 3 \ 3 \ 2 \ 4]
\]

\[
A' = [2 \ 3 \ 2 \ 3 \ 3 \ 2 \ 5] + [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -1] = [2 \ 3 \ 2 \ 3 \ 3 \ 2 \ 4]
\]

\[
B = [2 \ 3 \ 2 \ 3 \ 2 \ 2 \ 3 \ 2 \ 3 \ 2 \ 3 \ 2 \ 3 \ 2 \ 3 \ 2 \ 3 \ 2 \ 3 \ 2 \ 3 \ 2 \ 3 \ 2 \ 3 \ 2 \ 3 \ 2 \ 3 \ 2 \ 3]
\]

Fig. 15 shows the first measures of this simulation in traditional notation. In this version, the shortenings of certain durations of the rhythmic pattern have a slightly stronger effect than in comparison with the original piece.

In the second simulation, NSP=3, the number 9 is chosen to be decomposed into primes 2 and 7. This time, the more significant gap between the two base durations gives a different character to the first and third voices. The fixed additions to the rhythmic pattern have an even more substantial impact in this configuration. Complex rhythmic relationships arise in the simulation, as the 14:9 group in the fourth measure of the second voice (Fig. 16).
Fig. 16: First measures of a simulation using numbers 2, 7 and 9 to compose the cells of the rhythmic pattern.

As an example of testing the parameter with an extreme value, the piece was simulated with the NSP parameter at 50. Now the number 72 is selected to be decomposed into primes 5 and 67. The beginning of this slow, distorted version of the piece is represented in traditional notation in Fig. 17.

Note that the particular behavior of this simulation comes from the way the NSP parameter was implemented. A possibly better way would be to decompose the \( n \)th sum of primes not into any two different primes but the closest pair. Such a solution could allow for a more coherent space of variants.

Fig. 17: First measures of a simulation using numbers 5, 67 and 72 to compose the cells of the rhythmic pattern.

**Parameter: pitch-unit.** The pitch-unit parameter is independent of the gamut parameter. It will affect the calculation of the complement set, the pitches played by the left-hand voices, but will let all that is to be performed by the right hand unaffected.
In the test case for this parameter, we set the pitch-unit to 50 cents (a quarter-note) which implies a 24-temperament pitch-space. This time, the 7-note diatonic scale plugged into the gamut parameter yields a 17-note complement in place of the original 5-note one. In the resulting simulation, the diatonic vs. pentatonic dichotomy gives place to a kind of polarity between 12-tone diatonicism vs. a 24-tone tempered chromaticism. Fig. 18 illustrates the calculation of the complement for the B locrio scale for such a 24-tone pitch-space. Fig. 19 shows the first measures of this simulation, followed by the piano roll of the first section. The piano roll shows how the ambitus of the left-hand voices are significantly restrained, as there are many more pitches to cover inside the octave.

![Fig. 18: When changing the pitch-space from the 12-tone equal temperament to 24, the pitch-set of the right hand (the B locrio scale) yields a 17-note complement instead of a 5-note complement.](image)

![Fig. 19: First measures of a simulation using 50 cents as the pitch-unit.](image)

**Conclusions**

With this study, we have given an example of how structural features of a musical composition, namely György Ligeti’s first piano étude, can be reenacted by deterministic routines controlled by way of parameters. Manipulating parameters allowed the computational simulation of the piece within reach of its original premises and beyond. To study a musical score by means of modeling and simulation portends, to some extent, to revive its creative process. By doing so, we hope to have succeeded in not only raising the previous hypothesis anew but also shedding new light on such a pinnacle of the piano repertoire.
The model presented here could be improved and extended in many ways. For instance, it could be generalized to allow an arbitrary number of voices. That could be achieved by partitioning the pitch-space, particularly those more extensive than the twelve-tone equal temperament, into more than two pitch-sets. Also, the length of the inner cycles of the melodic voices could be controlled by a parameter. For this purpose, the elaboration of explicit formulae, ideally ones that take into account the rising sinuoid amplitude of their pitch profile, could be the most plausible strategy.

Concerning implementation, the algorithm for decomposing a number into two primes is not ideal and could be significantly improved. Furthermore, our implementation, as well as that of previous studies (KUNZE, 2003. TAUBE, 2013) deals with too many “hardcoded” series of degrees and durations. It would be more desirable to extend the model to algorithmically generate those series, even if done by decidedly arbitrary procedures. It should be addressed by a more encompassing, subsequent model.

Concerning the simulations, we were perhaps too conservative by not running the model with different, arbitrary series of degrees and durations, as well as different vectors to transform the underlying rhythmic patterns (cf. section “Durational perturbations and form”). Those hardcoded inputs could even be left to stochastic processes. As our goal was aimed more at exploring the simulations from an analytical/explanatory perspective, we were less focused on how the computer model could be used in entirely different contexts, i.e., whether it would be able to generate apparently unrelated compositions.

Finally, among the questions this study raises lies one in regard to how such algorithmic models could benefit performers and audiences in general. Future works should address features that could help performers to study and prepare the piece. In the same way, the analytic/explicative side of conceiving parameters and using them to run simulations should be placed at the service of the general public, constituting the ultimate goal of modeling and simulation focused on musical analysis.

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References


24 In the end of his report Kunze, says: “The temporal processes, however, contain a fair amount of handcoding and are thus less easily transformed, although possibilities exist to replace them with more formal techniques” (KUNZE, 2003, emphasis added).


Appendix 1: Algorithm 1 - high-level description of the global algorithm used to generate a section of Désordre.

```plaintext
procedure generate-complement:
  input: pitch-collection, pitch-unit
  convert pitch-collection to pitch-classes
  let chromatic-scale = chromatic scale built on pitch-unit
  let complement = set difference between chromatic-scale and pitch-classes
  convert complement to complement-pitch-collection
  rotate-left complement-pitch-collection, maintaining ascending order
```
procedure generate-base-rhythm:
  input: NSF, pattern
  initialize base-rhythm
  decompose the n-th sum of two primes (NSP) into a pair of different primes x, y
  for cell in pattern:
    when cell equals 'a', append (x,y) to base-rhythm
    when cell equals 'b', append (y,x) to base-rhythm
    when cell equals 'c', append (x+y) to base-rhythm
  return base-rhythm

procedure generate-foreground:
  input: pitch-collection, degree-list, perturbation-list, base-rhythm, doubling-interval
  initialize the empty lists melody-list and rhythm-list
  let number-of-cycles = length of perturbations-list
  for cycle 1, 0 <= i < number-of-cycles:
    transpose by i*transposition steps each degree of degree-list
    map degree-list onto pitch-collection and append to melody-list
    apply i-th perturbation to base rhythm and append to rhythm-list
  double resulting sequence of pitches in melody-list with doubling-interval
  return (melody-list, rhythm-list)

procedure generate-background:
  input: pitch-collection, degree-list, duration-list, beat-unit
  initialize the empty lists melody-list and rhythm-list
  for each degree n in degree-list with duration d in duration-list:
    map a run of d consecutive degrees starting in degree n onto
    pitch-collection and append to melody-list
    append d notes with duration beat-unit to rhythm-list
  return (melody-list, rhythm-list)

procedure generate-desordre-section:
  input: gamut, NSF, doubling-interval, beat-unit, pitch-unit
  hardcoded: degrees and rhythmic perturbations of foreground voices,
            degrees and durations of primitive version of background voices
  let RH = gamut
  let LH = generate-complement(RH, pitch-unit)
  let phrase-A = (a, a, b, c)
  let phrase-B-1 = (a, a, b, a, b, a)
  let phrase-B-3 = (a, a, b, a, a, a, b, c)
  let base-rhythm-1 = generate-base-rhythm(NSP, phrase-A + phrase-A + phrase-B-1)
  let base-rhythm-3 = generate-base-rhythm(NSP, phrase-A + phrase-A + phrase-B-2)
  let voice-1 = generate-foreground(RH, degree-list-1, perturbations-1, base-rhythm-1, 1, doubling-interval)
  let voice-3 = generate-foreground(LH, degree-list-3, perturbations-3, base-rhythm-3, -2, doubling-interval)
  let voice-2 = generate-background(RH, degree-list-2, duration-list-2, beat-unit)
  let voice-4 = generate-background(LH, degree-list-4, duration-list-4, beat-unit)
  return (voice-1, voice-2, voice-3, voice-4)
Appendix 2: First measures of Reconstruction
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